Cognitive Science 32 (2008) 445–457

ISSN: 0364-0213 print / 1551-6709 online
DOI: 10.1080/03640210701864147

Understanding and Using Principles of Arithmetic: Operations Involving Negative Numbers

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Received 9 November 2006; received in revised form 26 February 2007; accepted 14 March 2007

Abstract

Previous work has investigated adults' knowledge of principles for arithmetic with positive numbers (Dixon, Deets, & Bangert, 2001). The current study extends this past work to address adults' knowledge of principles of arithmetic with a negative number, and also investigates links between knowledge of principles and problem representation. Participants ($N = 44$) completed two tasks. In the Evaluation task, participants rated how well sets of equations were solved. Some sets violated principles of arithmetic and others did not. Participants rated non-violation sets higher than violation sets for two different principles for subtraction with a negative number. In the Word Problem task, participants read word problems and set up equations that could be used to solve them. Participants who displayed greater knowledge of principles of arithmetic with a negative number were more likely to set up equations that involved negative numbers. Thus, participants' knowledge of arithmetic principles was related to their problem representations.

Keywords: Psychology; Learning; Problem solving; Arithmetic; Conceptual knowledge

1. Introduction

Many models of problem solving include conceptual knowledge components such as principles. Principles can be defined as general rules that capture regularities within a domain. For example, one principle that applies in the domain of mixture problems is that the concentration of the final solution must be in between the concentrations of the two initial solutions. Problem solvers have been shown to use principles in a variety of domains, including counting (e.g., Gelman & Gallistel, 1978), proportional reasoning (e.g., Dixon & Moore, 1996), physics (e.g., Chi, Glaser, & Rees, 1982), and arithmetic (e.g., Dixon, Deets, & Bangert, 2001).

In the domain of arithmetic, prior research has focused on principles that underlie arithmetic operations. Dixon et al. (2001) identified principles that apply to operations involving

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positive numbers. Among others, these principles include *direction of effect* and *relationship to operands*. The direction of effect principle notes that increasing or decreasing the values of the operands affects the value of the result. For example, given an addition equation ($A + B = C$), increasing either operand (A or B) increases the value of the sum (C). Given a subtraction equation ($A - B = C$), increasing the subtrahend (B) decreases the value of the difference (C). The relationship to operands principle notes the relative values of the operands and result. For example, given an addition equation ($A + B = C$), the sum (C) must be greater than both operands (A and B). Given a subtraction equation ($A - B = C$), the difference (C) must be less than the minuend (A). A deep, conceptual understanding of arithmetic operations presumably involves knowledge of principles such as these.

Dixon et al. (2001) assessed knowledge of four principles for each of the four basic operations (addition, subtraction, multiplication, and division) with eighth-grade and college students. Participants viewed sets of sample problems that had been solved by hypothetical younger students; some sets included principle violations and others did not. Participants rated the level of understanding revealed in each set of solved equations. College students displayed knowledge of more principles than eighth-graders. Furthermore, students displayed knowledge of the greatest number of principles for addition, which is the earliest learned and most frequently used operation, and the fewest number of principles for division, which is typically the last learned of the basic operations. Thus, the results are consistent with the idea that principles are acquired as a result of experience with the operations (see also Dixon & Bangert, 2005).

The present study builds on Dixon and colleagues' research to consider adults' knowledge of principles for operations that involve negative as well as positive numbers. One goal of this work is to establish whether adults have knowledge of principles for operations with negative numbers. Even though people may be able to competently operate on negative numbers, they may not have conceptual knowledge of those operations. Indeed, many studies have shown that people can have procedural knowledge without the corresponding conceptual knowledge (e.g., Hiebert & Wearne, 1996; Rittle-Johnson & Alibali, 1999).

Past research suggests that both negative and positive numbers can be conceptualized in terms of a spatial mental number line (Dehaene, Bossini, & Giraux, 1993; Fischer, 2003; Fischer & Rottmann, 2005; Prather & Boroditsky, 2003). However, despite this similarity, we suggest that there is an important conceptual difference between positive and negative numbers—namely, negative numbers do not readily map to objects. Positive numbers can be used to quantify things in the real world, such as three apples or five pounds of grain. Negative numbers cannot be conceptualized in this way. One can imagine the number "3" as referring to three objects, but it is difficult to imagine "-3" in a similar way. This conceptual difference may not matter to mathematicians and high-ability individuals, who might rely on the spatial mental number line instead, but it may make a difference for less mathematically sophisticated individuals.

To formulate hypotheses about adults' knowledge of principles of arithmetic with negative numbers, we considered how people might acquire such knowledge. One possibility is that they might detect and extract regularities over the course of repeated exposure to operations on negative numbers, as seems to be the case for principles for operations with positive numbers (Dixon & Bangert, 2005; Dixon et al., 2001). On the face of it, however, this possibility seems unlikely, because exposure to operations on negative numbers is fairly minimal.

Table 1
Arithmetic principle definitions

Relationship to Operands	
Addition	*Positives* (A + B = C): The sum (C) must be greater than either operand (A or B)
	Positive + Negative (A + −B = C): The sum (C) must be smaller than the positive operand (A) and larger than the negative operand (−B)
Subtraction	*Positives* (A − B = C): The difference (C) must be less than the minuend (A)
	Positive − Negative (A − −B = C): The difference (C) must be greater than the minuend (A) and greater than the subtrahend (−B)
Direction of Effect	
Addition	*Positives* (A + B = C): Increasing either operand (A or B) increases the sum (C)
	Positive + Negative (A + −B = C): Increasing either operand (A or −B) increases the sum (C)
Subtraction	*Positives* (A − B = C): Increasing the minuend (A) increases the difference (C), increasing the subtrahend (B) decreases it
	Positive − Negative (A − −B = C): Increasing the minuend (A) increases the difference (C), increasing the subtrahend (−B) decreases it
Sign	
Addition	*Positives* (A + B = C): The sum (C) is always positive
	Positive + Negative (A + −B = C): The sum (C) is negative if the absolute value of B is greater than A, positive if the absolute value of B is less than A, and zero if the absolute value of B is equal to A
Subtraction	*Positive* (A − B = C): The difference (C) is negative if B is greater than A, positive if B is less than A, and zero if B is equal to A
	Positive − Negative (A − −B = C): The difference (C) is always positive

A second possibility is that individuals might transfer principles from operations with positive numbers, which they already understand. According to this hypothesis, for any given principle, individuals should display knowledge of that principle for operations with negative numbers only if they also display knowledge of that principle for operations with positive numbers. This hypothesis is compatible with the idea that negative numbers are more conceptually complex than positive numbers, perhaps due to the lack of object representation for negative numbers.

In this study, we consider adults' knowledge of principles for addition and subtraction with positive and negative numbers (see Table 1). We consider the direction of effect and relationship to operands principles (described above) as well as the *sign* principle. The sign principle specifies the sign (positive or negative) of the result based on the relative values of the operands. For example, in an equation of the form $A + -B = C$, the sum (C) will be negative if the absolute value of −B is greater than A and positive if it is not (zero if they are equal).

A second goal of this work is to investigate consequences of principle understanding. It seems likely that people who have strong principle knowledge will solve related problems more accurately than people who have weak principle knowledge. However, there may be other consequences as well. We suggest that knowledge of principles may also relate to problem representation. Problem representation can be defined as "the internal depiction or re-creation of a problem in working memory during problem solving" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 348). Past research has shown that individuals with greater conceptual knowledge represent problems more accurately than those with less conceptual knowledge (e.g., Chi, Feltovich, & Glaser, 1981; Rittle-Johnson et al., 2001).

There are many possible ways to represent arithmetic relations, some of which involve negative numbers and some of which do not. For example, $5 + -3 = x$ can also be represented as $5 - 3 = x$. In the present study, we investigate links between participants' principle knowledge and their representations of word problems that could involve arithmetic operations with negative numbers. We hypothesize that participants with greater knowledge of principles for operations with negative numbers will be more likely to use negative numbers in representing the problems than participants with less knowledge.

We tested participants' knowledge of principles of arithmetic with positive and negative numbers using the task developed by Dixon et al. (2001). We examined participants' representations of word problems by asking them to produce equations that they could use to solve the problems. Some possible correct equations involve negative numbers, but others involve only positive numbers. Finally, we examined whether participants' performance on the principle tasks was related to their representations of the word problems.

2. Method

2.1. Participants

Participants were 45 undergraduate students, recruited from a psychology department participant pool. They received extra credit in exchange for their participation. One participant withdrew before completing the word problem task. Data from this participant were included in the analyses where possible, so the *N* for some analyses is 45, and for others, 44.

2.2. Procedure

Participants took part individually in one experimental session that included three tasks, described below. The word problem task was always presented last, and the remaining two tasks were presented in counterbalanced order.

	3	**4**	**5**
16	16 – 3 = 20	16 – 4 = 19	16 – 5 = 18
17	17 – 3 = 21	17 – 4 = 20	17 – 5 = 19
18	18 – 3 = 22	18 – 4 = 21	18 – 5 = 20

How good or bad was this student's attempt at arithmetic?

Very Bad 1 2 3 4 5 6 7 *Pretty Good*

Fig. 1. Sample equation set as seen by participants in the evaluation task, including rating scale. The example is a violation of relationship to operands for subtraction with positive numbers.

2.2.1. Evaluation task

The evaluation task was based on that used by Dixon et al. (2001) to assess knowledge of principles. Participants viewed sets of solved arithmetic equations, presented one at a time on sheets of paper. Each set consisted of nine equations presented in a 3×3 matrix (see Fig. 1). Participants were told that each set had been solved by a "hypothetical student who is learning arithmetic." Participants were also told that all of the equations were incorrect, but that they might believe that some students made better attempts at arithmetic than others. Participants were asked to rate each attempt on a scale from 1 to 7, with 1 indicating *very bad* and 7 indicating *pretty good*. This task was not timed.

Forty-eight trials were generated using a 2 (operation: addition or subtraction) $\times$ 3 (arithmetic principle: relationship to operands, direction of effect, or sign), $\times$ 2 (number type: positive or negative) $\times$ 2 (trial type: violation, nonviolation) design. Trials were grouped and presented in the following order: addition with positive numbers, addition with a negative number, subtraction with positive numbers, and subtraction with a negative number. Within each group, trial order was random.

Each violation trial had a corresponding nonviolation "partner" to which it was compared. For example, a trial involving addition with positive numbers that violates relationship to operands was compared to a trial involving addition with positives that does not violate any principles. Trial pairs were constructed so that the average amount wrong (across all nine equations) in each member of the pair was the same. Therefore, any differences in ratings could not be due to the solutions in one trial being further from the correct answers than those in the corresponding trial.

For the sign principle for addition with positive numbers and subtraction with a negative number, it is impossible to create equations that violate the sign principle that do not also violate the relationship to operands principle. To address this issue, we created four additional trials (two for addition with positive numbers, and two for subtraction with a negative number) that violated only the relationship to operands principle, to be paired with these "double violation" trials. These trial pairs were also constructed so that the average amount wrong in each member of the pair was the same. Thus, trials with sign and relationship to operands violations were compared to trials with only relationship to operands violations. Any differences in ratings should be due to the sign violation.

There were also four practice trials at the outset of the session. Thus, the total number of trials was 56.

2.2.2. Verification task

Participants viewed solved arithmetic equations one at a time on a computer monitor, and were asked to judge whether each equation was correct (e.g., "$-19 + 3 = -16$") or incorrect (e.g., "$15 + 5 = 11$"). Presentation time was brief (1300 milliseconds) to make the task more challenging. Participants were asked to respond as quickly and accurately as they could, and both speed and accuracy were recorded. Incorrect trials included both principle violations and nonviolations, because we intended to use the data to generate an additional measure of participants' knowledge of principles. However, there were no systematic differences in speed or accuracy on violation and nonviolation trials and no systematic relations with performance on the other tasks. Therefore, data from this task were not considered further.

2.2.3. Word problem task

Participants were given eight word problems and were asked to generate equations they could use to solve the problems. For each problem, there were multiple equations that could be considered correct. Some of the values in the problems could be represented by negative numbers, although it was not necessary to use negative numbers to create a correct equation for any of the problems. One problem was excluded from analysis because it was worded ambiguously. Therefore, the maximum possible score on the word problem task was 7. The following is a sample problem.

> Jane's checking account is overdrawn by $378. This week she deposits her paycheck of $263 and writes a check for her heating bill. If her checking account is now overdrawn by $178, how much was her heating bill?

This problem could be represented by several equations that are mathematically equivalent, including "$-378 + 263 - x = -178$" and "$378 - 263 + x = 178$." Both equations would lead to the correct value for the heating bill (63), but only the first uses negative numbers. (See the Appendix for additional examples.)

2.2.4. Mathematics experience

Participants completed a questionnaire about their mathematics background. They were asked to list all college-level math courses they had taken. We assigned each student an experience score based on their most advanced course: 1 (none), 2 (geometry, algebra or pre-calculus), 3 (one or two semesters of calculus), or 4 (third semester calculus or beyond). The average score was 2.41.

3. Results

3.1. Did participants display knowledge of arithmetic principles?

We first asked whether participants displayed knowledge of the three target principles for operations involving positive numbers and operations involving a negative number. We calculated the average rating participants provided for violation and nonviolation sets on the evaluation task for each operation, number type, and principle. If participants have knowledge of a given principle, they should rate violation sets lower than nonviolation sets for that principle.

As seen in Fig. 2, at the group level, participants displayed knowledge of all three principles for addition with positive numbers, direction of effect, $t(44) = 5.14$, $p < .001$; relationship to operands, $t(44) = 3.37$, $p = .001$; and sign, $t(44) = 2.68$, $p = .01$. They also displayed knowledge of direction of effect for subtraction with positive numbers, $t(44) = 2.93$, $p = .005$. Participants did not display knowledge of any of the three principles for addition with a negative number, but they displayed knowledge of direction of effect and sign for subtraction with a negative number, $t(44) = 2.79$, $p = .007$, and $t(44) = 2.17$, $p = .03$, respectively.

We hypothesized that participants would have greater knowledge of principles involving positive numbers than those involving negative numbers, due either to greater experience

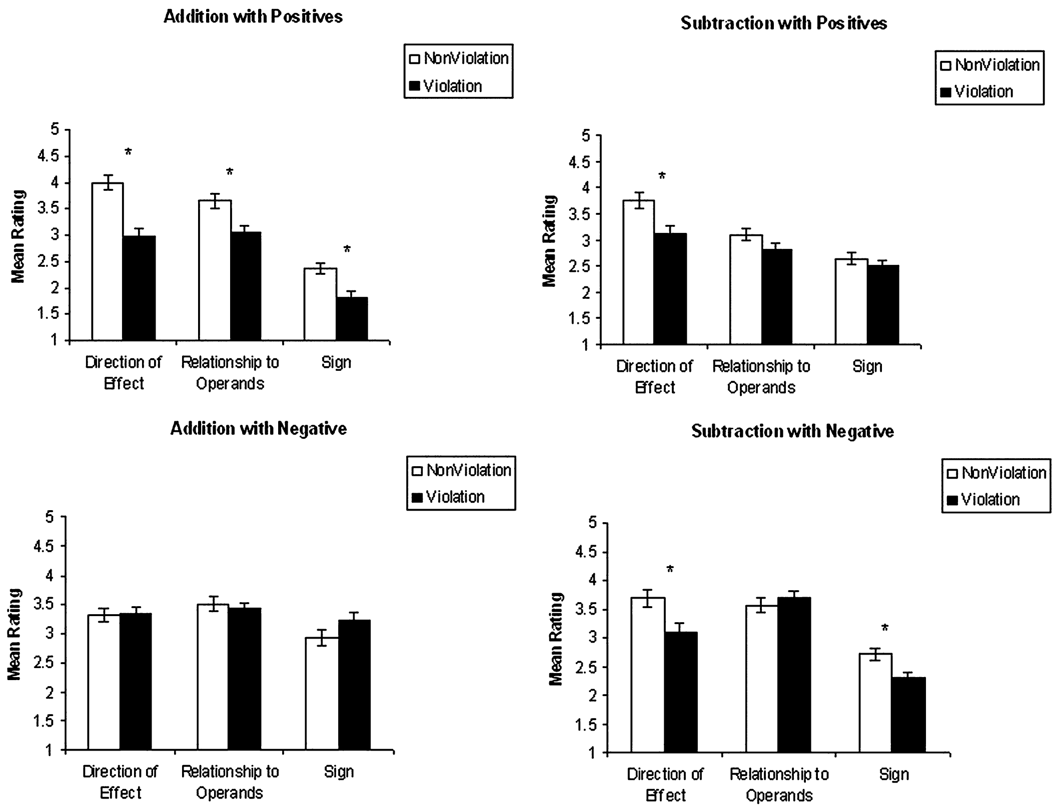

Fig. 2. Average ratings of nonviolation and violation sets as a function of principle, number type, and operation. The error bars represent standard errors. Asterisks indicate a significant difference between ratings of violations and nonviolations ($p < .05$).

with positive numbers, greater conceptual complexity of negative numbers, or both. To test this hypothesis, we calculated the difference between average ratings for violation and nonviolation sets for each participant for each operation, number type and principle. We then conducted a 3 (principle) $\times$ 2 (number type) $\times$ 2 (operation) repeated measures ANOVA, with difference between average ratings of nonviolation and violation trials as the dependent measure.

As expected, the main effect of number type was significant, $F(1, 44) = 31.22$, $p < .01$. On average, the difference between nonviolation and violation trials was greater for arithmetic with positives (.53) than for arithmetic with a negative (.11). However, this main effect was qualified by a significant interaction between operation and number type, $F(1, 44) = 13.32$, $p = .001$. As seen in Fig. 3, the average difference between nonviolation and violation trials was higher for addition with positive numbers than for addition with a negative number. However, the average difference between nonviolation and violation trials was comparable for subtraction with positive numbers and for subtraction with a negative number. Thus, in the case of addition, participants had greater principle knowledge when the operation involved positive numbers than when the operation involved a negative number; in the case of subtraction, there

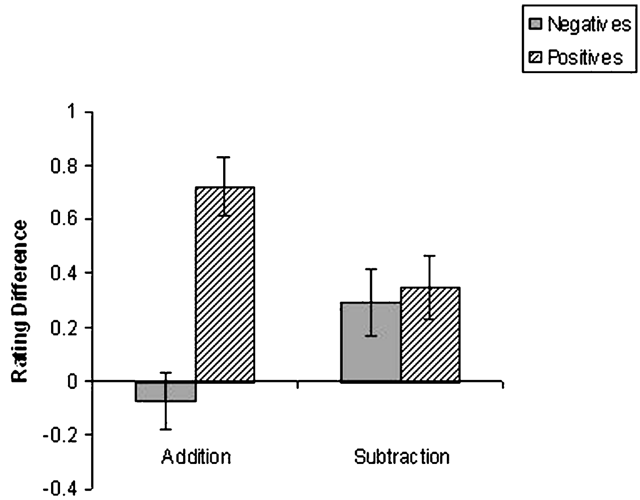

Fig. 3. Participants' average rating difference (nonviolation minus violation) on the evaluation task as a function of principle and number type. The error bars represent standard errors.

was no difference. Follow-up t-tests indicated that this pattern (significant effect of number type for addition, no effect of number type for subtraction) held for each of the three principles: relationship to operands: addition, $t(44) = 2.31$, $p = .02$, subtraction, $t(44) = 1.83$, $p = .07$; direction of effect: addition, $t(44) = 4.05$, $p < .001$, subtraction $t(44) = .15$, $p = .87$; sign: addition, $t(44) = 3.51$, $p = .001$, subtraction, $t(44) = 1.14$, $p = .25$.

The analysis also revealed a main effect of principle, $F(2, 88) = 3.83$, $p = .02$. On average, the difference between violation and nonviolation trials was greater for direction of effect (.55) than for relationship to operands (.21) or sign (.20).

3.2. *Knowledge of principles and problem representation on the word problem task*

We next asked whether participants' knowledge of arithmetic principles was associated with their representation of word problems. We hypothesized that participants with greater knowledge of principles for operations with a negative number would be especially likely to use negative numbers in representing the word problems.

To address this issue, we first calculated a measure of each individual's understanding of principles. Participants were assigned one point for each pair of trials on the evaluation task for which they rated the nonviolation set higher than the corresponding violation set. The highest possible score for each number type (positive and negative), summing across principles and operations, was 12. Mathematics experience was not significantly correlated with positive evaluation scores, $r = .18$, $z = 1.18$, and was marginally correlated with negative evaluation scores, $r = .24$, $z = 1.53$, $p = .06$, one-tailed.

For each participant, we then calculated two scores to capture performance on the word problem task: (a) the number of problems for which the participant produced a correct equation that involved a negative number ($M = 4.52$) and (b) the number of problems for

Table 2
Correlation matrix ($N = 44$)

	Experience	Positive Evaluation Score	Negative Evaluation Score	Word Problem: No. Correct	Word Problem: No. Correct w/ Negative Number
Experience	1.00				
Positive evaluation score	0.18	1.00			
Negative evaluation score	0.24	0.49**	1.00		
Word problem: no. correct	0.11	0.13	0.18	1.00	
Word problem: no. correct w/ negative number	0.12	0.40**	0.46**	0.69**	1.00

Note. $^{*}p < .05$, $^{**}p < .01$, all two-tailed.

which the participant produced a correct equation of any kind (i.e., negative numbers or not; $M = 5.29$). We then used multiple regression to examine predictors of performance on the word problem task. The predictors tested were (a) mathematics experience; (b) knowledge of principles for operations with a negative number, as assessed by negative evaluation score; and (c) knowledge of principles for operations with positive numbers, as assessed by positive evaluation score. A complete correlation matrix for these measures is presented in Table 2.

We first considered the number of word problems for which participants generated correct equations that involved a negative number. The sole significant predictor of this outcome measure was negative evaluation score, $\beta = .25$, $t(42) = 2.15$, $p = .038$. Neither positive evaluation score, $\beta = .16$, $t(42) = 1.43$, $p = .16$, nor mathematics experience, $\beta = .003$, $t(42) = .014$, $p = .99$, was significant. The overall R^2 value for the regression was .25. Thus, as hypothesized, knowledge of principles of arithmetic with a negative number was associated with use of negative numbers in representing the word problems.

We next considered the number of word problems for which participants generated a correct equation of any type (i.e., negative numbers or not). In this analysis, none of the predictors were significant: negative evaluation score, $\beta = .09$, $t(42) = 0.75$, $p = .46$; positive evaluation score, $\beta = 0.034$, $t(42) = 0.29$, $p = .77$; experience, $\beta = .088$, $t(42) = .41$, $p = .68$. The overall R^2 value for this regression was .037.

Thus, negative evaluation scores were the sole significant predictor of use of negative numbers in representing the word problems but were not associated with overall success at the word problems. Knowledge of principles of arithmetic with a negative number is related specifically to use of representations involving negative numbers in the word problem task and not just to participants' overall level of success.

4. General discussion

This study investigated adults' knowledge of principles for addition and subtraction involving positive and negative numbers, and examined whether this knowledge relates to their representations of word problems that could involve negative numbers. Two of the arithmetic principles investigated in this study—relationship to operands and direction of effect—were previously studied for addition and subtraction with positive numbers by Dixon et al. (2001). Our results replicate Dixon et al.'s finding in every case except for relationship to operands for subtraction with positive numbers. In that case, we did not find a significant difference in ratings of violation and nonviolation trials, whereas Dixon et al. did.

We extended Dixon et al.'s past work to show that adults also have knowledge of some principles for operations with a negative number. Specifically, we found that adults displayed knowledge of the direction of effect principle and the sign principle for subtraction with a negative number.

We expected that participants would display more knowledge overall for principles for operations with positive numbers than for principles for operations with a negative number, due to greater experience with positive numbers, greater conceptual complexity of negative numbers, or both. This pattern held for addition but not for subtraction. Thus, it was not the case that performance was always better for arithmetic with positive numbers than for arithmetic with a negative number. We suggest that, in reasoning about operations with negative numbers, participants may draw on knowledge about inverse operations with positive numbers. Thus, when viewing an equation that involves subtracting a negative number, participants may mentally "convert" to the equivalent operation of adding a positive number. This would explain why we found that knowledge about subtraction with a negative number is relatively strong (overall, participants displayed knowledge of two principles for subtraction with a negative)—because it is converted to the well-understood operation of addition with positive numbers. However, knowledge of addition with a negative number may not benefit from conversion to subtraction with positives, because subtraction with positives is not especially well understood itself (overall, participants displayed knowledge of only one principle for subtraction with positive numbers). Future research should be designed to directly investigate whether participants mentally convert operations with negatives to inverse operations with positives.

Our second goal was to examine links between participants' knowledge of arithmetic principles and their representations of word problems. Knowledge of principles for operations with a negative number was the strongest predictor of use of correct equations with negative numbers to represent the word problems. However, knowledge of principles for operations with a negative number was not a significant predictor of use of correct equations of any kind to represent the word problems. This pattern suggests that principle knowledge is associated with how participants represent problems (either with an equation that involves only positive numbers or with an equation that contains a negative number). This goes beyond more knowledge predicting better performance—the issue is not whether participants generated a correct equation, but the kind of equation they generated. These findings suggest that principle knowledge guides problem representation. As such, they complement other research addressing the role of conceptual knowledge in problem solving (e.g., Rittle-Johnson et al., 2001). More generally, the findings highlight the interplay of different forms of knowledge in problem solving and arithmetic performance.

Some limitations of the present study must be acknowledged. First, we considered only three arithmetic principles: relation to operands, direction of effect, and sign. There are other regularities that characterize arithmetic operations that we have not tested. Second, we considered only arithmetic operations that involved a positive and a negative operand. Principles for operations involving two negative operands remain to be tested in future work. Third, our measure of mathematics experience was admittedly coarse. We did not find strong links between experience and performance on the word problems or between experience and principle scores; however, with a more sensitive measure of experience, such associations might be found.

Further work is needed to more fully describe people's knowledge of arithmetic operations, how this knowledge is applied and how it affects performance on problem-solving tasks. The current results show that principle knowledge is associated with problem representation, but they do not reveal how broadly participants' knowledge of principles may be applied. This study involved rating symbolic equations that had purportedly been completed in an academic setting. Would participants have performed similarly if problems had been presented in other contexts, such as in verbal or concrete form? Previous studies have shown that performance on some types of mathematical tasks is context dependent (e.g., McNeil & Alibali, 2005; Saxe, 1991). Knowledge of principles may be context dependent as well. This issue is an important arena for future work.

In sum, this study investigated adults' knowledge of principles for operations with negative numbers, which have received little attention in the cognitive science literature. This study showed that college students possess knowledge of some principles for arithmetic operations with a negative number. Specifically, they have knowledge of the direction of effect and sign principles for subtraction with a negative number. Overall, participants had greater principle knowledge for addition with positive numbers than for addition with a negative number; however, this was not the case for subtraction. The pattern of data suggests that participants may draw principle knowledge for operations with a negative number from corresponding knowledge for inverse operations with positive numbers. More generally, the results suggest that the representation of operations on negative numbers is substantially different from the representation of those same operations with positive numbers.

This study is also the first to document a relationship between principle understanding and problem representation. Specifically, adults who have greater knowledge of principles for operations with negative numbers were more likely to use negative numbers in representing word problems where their use is optional. This finding suggests that principle knowledge guides problem representation and, as such, it contributes to the growing body of work documenting the role of conceptual knowledge in mathematical problem-solving. Overall, the findings highlight the importance of principles as a form of conceptual knowledge that is integral in representing and solving problems.

Acknowledgments

This research was supported by a fellowship from the APA Minority Fellowship Program to R. W. Prather and by a grant from the National Science Foundation (BCS-0096129) to M. W. Alibali. Both authors acknowledge the members of the Cognitive Development Research Group for helpful feedback. We also thank James A. Dixon, Alexa Romberg, and the anonymous reviewers for helpful comments on a previous version of the manuscript.

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Appendix: Sample word problems and sample participant responses

Jane's checking account is overdrawn by \$378. This week she deposits her paycheck of \$263 and writes a check for her heating bill. If her checking account is now overdrawn by \$178, how much was her heating bill?

$$-378 + 263 - x = -178$$

$$378 - 263 + x = 178$$

$$263 - x = 378 - 178$$

A climbing team starts at the foot of a mountain that is 128 feet below sea level. They climb up 3282 feet and spend the night. The next day they climb back down 1845 feet to pick up forgotten supplies at a base camp. What is the altitude of the base camp relative to sea level?

$$-128 + 3282 - 1845 = x$$

$$3282 - 1845 - 128 = x$$

$$3282 - 1845 - x = 128$$