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Knowledge of Mathematical Equivalence in Children With Specific Language Impairment: InsightsFrom Gesture and Speech

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Abstract

Purpose—This study investigated understanding of mathematical equivalence in children with and without specific language impairment (SLI).

Method—A total of 34 children (ages 8;1 [years;months] to 11;7), including 9 with expressive SLI (E-SLI), 8 with expressive and receptive SLI (ER-SLI), and 17 age-matched typically developing (TD) children completed addition and mathematical equivalence problems. The problem-solving strategies revealed in solutions and in gestural and verbal explanations were coded.

Results—The children with SLI were less accurate than their TD peers in solving addition and equivalence problems. None of the children in the ER-SLI group solved the equivalence problems correctly; however, the number of children who solved any of the equivalence problems correctly did not differ in the E-SLI and TD groups. Children in the ER-SLI group tended to express incorrect strategies for solving the equivalence problems in both gesture and speech, whereas children in the E-SLI group often expressed *correct* strategies in gestures, but *incorrect* strategies in speech.

Conclusion—Children with SLI showed delays in their knowledge of mathematical equivalence. Children with ER-SLI displayed greater delays than children with E-SLI. Children with E-SLI sometimes expressed more advanced knowledge in gestures, suggesting that their knowledge is represented in a nonverbal format.

Keywords

SLI; mathematical equivalence; E-SLI; ER-SLI; gesture

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The purpose of this study was to investigate understanding of mathematical equivalence problems (i.e., equations with addends on both sides, such as 3 + 4 + 6 = 3 + __) in children with specific language impairment (SLI) and age-matched typically developing (TD) peers using information that children express in their gestures and speech. The majority of research on SLI has focused on the language impairments seen in these children (for detailed review, see Bishop, 1997; Leonard, 1998), but there is a growing body of work showing that children with SLI have deficits in mathematical abilities as well, often continuing into adulthood (e.g., Arvedson, 2002; Conti-Ramsden, Donlan, & Grove, 1992; Donlan, 2003; Fazio, 1994, 1996, 1997, 1999).

Mathematical Knowledge in Children With SLI

To date, investigations of mathematical knowledge in children with SLI have focused primarily on tasks such as magnitude comparison, one-to-one correspondence, rote counting, and arithmetic calculation. The present study investigated children's knowledge of mathematical equivalence, which is the idea that the two sides of an equation represent the same quantity. Children's understanding of mathematical equivalence is evidenced in successful solutions to problems such as 3 + 4 + 6 = 3 + __ and 4 + 5 + 3 = ___ + 5 (e.g., Perry, Church, & Goldin-Meadow, 1988).

Children with SLI do not exhibit deficits across all mathematical domains. Specifically, studies of one-to-one correspondence and magnitude comparison have shown that children with SLI have comparable accuracy (though slower performance) relative to TD peers. For example, Conti-Ramsden et al. (1992) presented 7-year-old children with SLI and TD peers with random arrays of two to nine dots and asked them to touch each dot as they counted aloud. Children with SLI took longer to count the dots, but they were as accurate as their TD peers. This finding was replicated by Donlan (1993) with slightly older children. Likewise, in a paradigm known as magnitude comparison, which is designed to tap individuals' abilities to judge the relative values of number symbols (e.g., Moyer & Landauer, 1967), children with SLI performed as accurately as, although more slowly than, TD peers (Donlan, Bishop, & Hitch, 1998; Siegel, Lees, Allan, & Bolton, 1981). These studies suggest that children with SLI have appropriate concepts of one-to-one correspondence as well as intact concepts of number magnitude.

In other mathematical domains, such as sequential counting and arithmetic, however, children with SLI have shown significantly impaired performance relative to TD peers. For example, in a series of longitudinal studies investigating changes in counting and arithmetic skills in children with SLI, Fazio (1994, 1996) observed that children with SLI around 5 years of age could rote count only up to six, whereas their TD peers could rote count up to 20 and beyond. The errors made by children in the SLI group were primarily sequencing errors, or number words that were out of order in the sequence. In a follow-up study 2 years later, the same children continued to be significantly impaired in rote counting relative to their TD peers (Fazio, 1996). Arvedson (2002) found that the counting abilities of children with SLI were significantly poorer than those of their TD peers but better than those of younger grammar-matched peers.

Children with SLI have also shown marked difficulties in their ability to solve arithmetic problems correctly. For example, Fazio (1996) compared the performance of 6- and 7-year-old children with SLI and TD peers on simple addition and subtraction problems. Children with SLI not only made significantly more calculation errors than their TD peers, but they also used developmentally less mature “count all” strategies, whereas children in the TD group tended to use developmentally more advanced “count from the larger number” strategies. In a follow-up study when the children were 9 and 10 years old, Fazio investigated their addition, subtraction, multiplication, and division skills in more depth (Fazio, 1999). Across all problem types, the children with SLI performed less well than their TD peers, making more calculation errors and procedural missteps. Further, the children with SLI used developmentally less mature, counting-based strategies in contrast to the more advanced strategies used by children in the TD group, such as retrieving correct answers from memory or using a related fact.

Several hypotheses have been proposed to account for the mathematical difficulties seen in children with SLI. Fazio (1994, 1996) considered two types of potential difficulties: (a) poor language abilities needed for the sequential procedure of counting (e.g., producing number words in the correct order), and (b) difficulties with the symbolic, conceptual aspects of counting. This latter possibility is based on the idea that mathematical notation is, in many ways, a language system. In mathematics, an arbitrary symbol (e.g., 2, >, +) is assigned to some concept (e.g., a quantity, a “greater than” relation, the operation of addition) and is then manipulated in conjunction with other symbols (e.g., $1 + 2 > 2$). This parallels the way language works. An arbitrary symbol (e.g., the English word *boy*, the Spanish word *niño*, or the American Sign Language sign that looks similar to tipping a hat) is assigned to a concept (e.g., male child) and is manipulated in conjunction with other words or signs. If this conceptualization of mathematics as an abstract language-like system is correct, it may be that the mathematical deficits seen in children with SLI are secondary to their language impairments. This argument is similar to that made by Nunes and Bryant (1996), who claimed that language abilities are critical to the development of mathematical skills in children with typical development. Along similar lines, Sophian (2007) distinguished between the “mathematics of quantities,” which involves knowledge about physical quantities and relations among them, and the “mathematics of symbols,” which involves knowledge about arbitrary symbols, such as numbers. It seems that children with SLI may have special difficulty with the mathematics of symbols.

Studies investigating mathematical skills in children with SLI have generally included either children with predominantly expressive deficits (E-SLI) or children with a mix of expressive and receptive deficits (ER-SLI). In domains other than mathematics, studies of expressive and receptive skills in children with SLI have suggested that children with E-SLI may differ qualitatively from children with ER-SLI (e.g., Aram & Nation, 1975; Rapin & Allen, 1983; Wolfus, Moskovitch, & Kinsbourne, 1980). In particular, children with E-SLI appear to exhibit good phonological discrimination and good semantic and syntactic comprehension skills, but they perform poorly on a range of expressive language formulation tasks. In contrast, children with ER-SLI exhibit deficits in both comprehension and production of language, and they may be slower in processing language (e.g., Conti-Ramsden & Durkin, 2007; Craig & Evans, 1993; Evans, 1996; Evans & MacWhinney, 1999; Evans, Viele, & Kass, 1997; Leonard, Nippold, Kail, & Hale, 1983; Montgomery, Scudder, & Moore, 1990).

In the area of mathematics, Fazio's (1994, 1996, 1999) studies investigating number concepts and mathematical skills in children with SLI included primarily children with predominantly expressive deficits and relatively intact receptive skills. For example, the 4- and 5-year-old children with SLI in Fazio's original (1994) study had primarily expressive language impairments, with expressive language abilities below the 10th percentile on the Developmental Sentence Analysis (Lee, 1966) but receptive language abilities no more than 6 months below age-level expectations, as measured by the Auditory Comprehension subtest of the Preschool Language Scale—Revised (Zimmerman, Steiner, & Pond, 1979). In the 2-year follow-up study in this series (Fazio, 1996), the children with SLI still had primarily expressive language impairments, with Listening Quotient scores > 1 *SD* below the mean and Spoken Language Quotient scores < 1 *SD* below the mean, as measured by the Test of Language Development 2—Primary (Newcomer & Hammill, 1988). In the final study in this series (Fazio, 1999), the expressive and receptive language abilities of the children with SLI were both only mildly impaired, with Expressive Language scores (ELSs) averaging 81 (*SD* = 3.0) and Receptive Language scores (RLSs) averaging 83 (*SD* = 4.2), as measured by the Clinical Evaluation of Language Fundamentals—Revised (CELF–R; Semel, Wiig, & Secord, 1989).

In the magnitude comparison study carried out by Donlan, Bishop, and Hitch (1998), the children with SLI all had expressive language scores falling < 1.5 *SD*s below the mean, as measured by the Action Picture Test (APT; Renfrew & Hancox, 1999). The receptive language abilities of the children with SLI in the study varied, however, with three of the 10 children having receptive language scores on the Test for Reception of Grammar (Bishop, 1982) falling < 1.5 *SD*s below the mean and the remaining seven children with SLI having receptive language scores > 1.5 *SD*s from the mean.

Thus, prior studies of mathematical knowledge in children with SLI have used heterogeneous samples that included both children with E-SLI and children with ER-SLI. Given the heterogeneity in language abilities of the children with SLI in prior studies, and the question of the degree to which language abilities may be critical to the development of mathematical skills (e.g., Nunes & Bryant, 1996), the language abilities of children with SLI need to be taken into consideration before any conclusions can be made regarding mathematical deficits in children with SLI.

Assessing Mathematical Knowledge Using Children's Gestured and Spoken Problem Explanations

In studies of mathematical reasoning in children with typical development, children's knowledge is often conceptualized in terms of shifts in the *strategies* they use to solve mathematical problems, for example, shifts from counting-based to retrieval-based strategies. Older children generally use more accurate and more efficient problem-solving strategies for arithmetic and algebraic equations than do younger children (e.g., Knuth, Stephens, McNeil, & Alibali, 2006; Lemaire & Siegler, 1995). Moreover, children with typical development sometimes express newly emerging strategies in their gestures in responses termed “gesture–speech mismatches” (Church & Goldin-Meadow, 1986; Perry et al., 1988) before they are able to express those strategies in speech (Alibali & Goldin-Meadow, 1993). It has been suggested that the strategies expressed uniquely in a child's gestures are not explicit or available to the child in a verbal format (Alibali & Goldin-Meadow, 1993; Garber, Alibali, & Goldin-Meadow, 1998). Children's spontaneous gestures may reflect implicit knowledge that is based in perception or action and is represented in a visual or motoric format as opposed to a symbolic or verbal format (Goldin-Meadow & Alibali, 1995).

Two previous studies investigated the spoken and gestured strategies expressed by children with SLI in their explanations of Piagetian conservation tasks (Evans, Alibali, & McNeil, 2001; Mainela-Arnold, Evans, & Alibali, 2006). Evans et al. (2001) found that, when children with SLI produced gestures in their explanations of the conservation tasks, they were more likely to convey different strategies in their gestures and speech compared to peers with typical development who offered the same pattern of same and different judgments across the tasks. Moreover, the children with SLI often conveyed more *advanced* understanding of conservation in their gestures than in their speech. Church and Goldin-Meadow (1986) observed a similar pattern among children with typical development who were on the brink of learning to conserve. One interpretation of Church and Goldin-Meadow's findings is that children whose knowledge is “transitional” simultaneously activate multiple strategies, including ones that are represented in a visual or motoric format.

Gesture–speech data have also been used to study the acquisition of mathematical equivalence in children with typical development (e.g., Perry et al., 1988). This work suggested that children traverse a three-step developmental path in acquiring the concept of mathematical equivalence (Alibali & Goldin-Meadow, 1993). They start out by considering a single, incorrect strategy for solving equivalence problems. This incorrect thinking is manifested in explanations in which children express the same, incorrect strategy in both gestures and speech (gesture–speech matches). As children's understanding of equivalence develops, they progress into a transitional period during which they frequently express different strategies in gestures and speech (i.e., “gesture–speech mismatches”). Finally, children begin considering a single, correct strategy for each problem, manifested in explanations in which children express the same, correct strategy in both gestures and speech.

Looking at children's position on this developmental path reveals more subtle information about their understanding of equivalence than their solution accuracy reveals alone. For example, two children might solve all of the problems incorrectly, but one might produce primarily explanations in which the information conveyed in both gestures and speech is incorrect. In contrast, the second child might produce primarily mismatch explanations in which incorrect strategies are conveyed in speech but correct strategies are conveyed in gestures. Prior work with children with typical development suggests that this second child has a more advanced understanding of the concept, and importantly, is more “ready” to learn from direct instruction (Alibali & Goldin-Meadow, 1993; Perry et al., 1988).

In our research, we considered information that children expressed in gestures and speech to gain insight into their understanding of mathematical equivalence. In particular, although children with SLI may appear to have poor understanding of mathematical equivalence, it may be that their knowledge about mathematical equivalence is represented in a nonverbal, perceptual format, and it may be reflected in strategies that are expressed uniquely in gestures, in gesture–speech mismatches. Thus, as in the conservation study (Evans et al., 2001), it may be the case that children with SLI express more advanced strategies in gestures than in speech.

Present Study

The purpose of the present study, therefore, was to investigate knowledge of mathematical equivalence in children with SLI and TD peers. Based on prior work suggesting that children with SLI may have intact number concepts but deficits in computational skills, we hypothesized that children with SLI may have an appropriate *concept* of mathematical equivalence, so they may attempt to use correct strategies to solve equivalence problems (e.g., making both sides of the equation sum to the same total). However, they may also encounter difficulties due to their weak computational skills. We hypothesized that considering the information children express in gestures may reveal that children with SLI have knowledge about mathematical equivalence that is represented in a non-verbal, perceptual format and that is manifested in gesture–speech mismatches, particularly ones in which gestures reveal more advanced strategies than speech.

Further, given the potential links between mathematical abilities and language abilities, we hypothesized that children with E-SLI may have knowledge of mathematical equivalence that is more advanced than that of children with ER-SLI. Specifically, we hypothesized that children with E-SLI may have intact knowledge about mathematical equivalence; however, due to their expressive deficits, they may tend to express this knowledge in gestures and not in speech. Children with ER-SLI, on the other hand, may have less advanced knowledge of mathematical equivalence, so they will be unlikely to express advanced knowledge of equivalence in gestures or in speech.

In sum, then, we addressed the following four research questions:

- Do children with SLI show impaired accuracy on addition and equivalence problems relative to children with typical development?
- Do children with SLI use different strategies than children with typical development to solve equivalence problems?
- Are children with SLI delayed in their acquisition of equivalence, as manifested in their gestured and spoken problem explanations, compared to TD peers?
- Are children with SLI, and in particular, children with E-SLI, especially likely to express more advanced knowledge about equivalence in their gestures than in their speech?

METHOD

Participants

Participants were 17 children with SLI (ages 8;1 [years;months] to 11;7) and 17 TD children with typical development (ages 8;1 to 11;6). A group-wise matching was conducted by randomly drawing children from a larger pool of children participating in gesture studies (e.g., Mainela-Arnold et al., 2006) until the groups were matched by age, $F(2, 31) = .12$, $p > .05$, $\eta^2 = .01$. Participants were recruited from public and parochial schools in a midsized city in the Midwest. All were part of a larger study investigating gesture in children with SLI. All children met the following criteria: (a) Nonverbal Intelligence Quotient (IQ) ≥ 85 as measured by the Leiter International Performance Scale—Revised (Roid & Miller, 1997), the Test of Nonverbal Intelligence, Third Edition (Brown, Sherbenou, & Johnsen, 1997), or the Columbia Mental Maturity Scale (Burgemeister, Blum, & Lorge, 1972); (b) normal hearing (pure tone screening at 500 Hz, 1, 2, and 4 kHz at 20 dB); (c) normal or corrected vision; (d) normal oral and speech motor abilities; (e) speech intelligibility ≥ 90% as measured by the Arizona Articulation Proficiency Scale (Fudala & Reynolds, 1989); and (f) residence in a monolingual, English-speaking home. Children were not eligible to participate if they had any of the following conditions based on parent report of prior medical, psychological, or educational assessments: (a) mental retardation; (b) emotional, psychological, or behavioral disturbances; (c) motor deficits or frank neurological signs; or (d) seizure disorders or use of medications to control seizures. Attention deficit hyperactivity disorder was not considered an exclusionary criterion for participation. However, on the parent report, none of the parents indicated that their child was taking medication for attention deficit hyperactivity disorder.

Children's language abilities were assessed using the CELF–R. The children with SLI completed all subtests of the CELF–R, and the TD children completed all expressive subtests and the Oral Directions receptive subtest to screen for normal receptive language skills.

Children's knowledge of mathematics was assessed using the Arithmetic subtest of the Kaufman Assessment Battery for Children (K-ABC; Kaufman & Kaufman, 1983).[1] This subtest includes items that involve numeration and computation. An example test item includes a picture of elephants and a description of one elephant weighing 650 pounds and another elephant weighing 550 pounds. The child is asked: “How much more does the big elephant weigh than the small one?”

A total of 19 children with SLI participated in the study. However, one child with SLI was excluded due to videotape failure. A second child showed lack of understanding of simple arithmetic problems (e.g., for the problem 2 + 4 + 5 + 2 =__, offered a solution of 58) and was therefore excluded from the sample.[2] Results of standardized language assessments for the remaining 17 children with SLI and their TD peers are shown in Table 1.

[1]Children completed testing before publication of the revised version of the Kaufman ABC-II.

[2]This child was from the ER-SLI group. Her performance was qualitatively different from that of other children with SLI in that her incorrect solutions were nonsensical and did not reflect computation errors.

To address the question of mathematical abilities and subgroup differences in SLI, the children in the SLI group were further classified into those having primarily expressive language impairments (E-SLI) and those having both expressive and receptive language impairments (ER-SLI), based on Evans' (1996) criteria. Several past studies have used these subgrouping criteria and have shown differences between the subgroups on various language tasks (e.g., Bishop & Donlan, 2005; Bishop, Hardiman, Uwer, & von Suchodoletz, 2007; Conti-Ramsden & Durkin, 2007; Craig & Evans, 1993; Evans & MacWhinney, 1999; Evans et al., 1997). However, no studies of children's mathematical knowledge to date have compared the performance of these subgroups. Children were designated as E-SLI if their (a) composite Receptive Language score on the CELF–R was ≥ 1 *SD* below the mean of 100, (b) composite Expressive Language score on the CELF–R was < 1 *SD* below the mean of 100, and (c) the difference between their Expressive and Receptive composite score was ≥11 points. Children were designated as ER-SLI if their (a) composite Receptive Language score on the CELF–R was < 1 *SD* below the mean of 100, (b) composite Expressive Language score on the CELF–R was < 1 *SD* below the mean of 100, and (c) the difference between their Expressive and Receptive composite score was ≤10 points.[3] According to these criteria, nine of the children were classified as E-SLI, and eight children were classified as ER-SLI.

There were differences among the three groups of children in nonverbal IQ, $F(2, 31) = 6.08$, $p < .05$, $\eta_p^2 = .28$ (i.e., 28% of the variation in IQ was attributable to group membership); Expressive Language score, $F(2, 31) = 51.12$, $p < .05$, $\eta_p^2 = .77$; receptive language as measured by the Oral Directions subtest of the CELF–R, $F(2, 31) = 20.17$, $p < .05$, $\eta_p^2 = .57$; and arithmetic skills as measured by K-ABC standard scores, $F(2, 31) = 24.10$, $p < .05$, $\eta_p^2 = .61$. The E-SLI and TD groups differed significantly in expressive language but did not differ significantly in nonverbal IQ, receptive language, or arithmetic knowledge. The ER-SLI and TD groups differed significantly in nonverbal IQ, expressive language abilities, receptive language abilities, and arithmetic knowledge. The E-SLI and ER-SLI groups also differed significantly in nonverbal IQ, expressive language abilities, receptive language abilities, and arithmetic knowledge.

To ensure that the children understood the nonmathematical verbal concept of equality, each child was required to complete a series of Piagetian conservation tasks. These tasks involve an initial verbal judgment of whether two quantities are equal (Mainela-Arnold et al., 2006). All of the children showed an understanding of equality of liquid, mass, length, and number.

Tasks

All of the children completed nine mathematics problems. The problems included three addition problems of the form $a + b + c + a =$ __ and six equivalence problems, three of the form $a + b + c = a +$ __, and three of the form $a + b + c =$ __ $+ c$. Addends for all of the problems were 6 or smaller (see the Appendix). Problems were presented on laminated cards that were clipped to a large easel. Children used a marker to write their solutions to the

[3]One child in the ER-SLI group had an Expressive Language score of 86 and a Receptive Language score of 76.

problems. After solving each problem, children gave the marker back to the examiner (to free their hands). Children were then asked to explain how they got their solutions. Children's solutions and explanations were both audio and video recorded. All children solved the problems in the same order.

Coding

Coding solutions—Children's problem solutions were classified as *correct* or *incorrect*. For equivalence problems, children's incorrect solutions were further categorized into one of three categories: those that resulted from adding all of the numbers (add all; e.g., for the problem 3 + 4 + 6 = 3 + __, a solution of 16), those that resulted from adding the numbers before the equal sign (add to equal sign; e.g., for the problem 3 + 4 + 6 = 3 + __, a solution of 13), and other incorrect solutions (e.g., for the problem 3 + 4 + 6 = 3 + __, a solution of 4). Add-all and add-to-equal-sign errors indicate common misconceptions about the meaning of the equal sign (i.e., the equal sign means *the total* or *put the answer*; see Rittle-Johnson & Alibali, 1999).

Coding gestured and verbal strategies—Children's verbal explanations and the accompanying gestures were transcribed and coded independently on separate passes through the video. Verbal explanations were coded with the video turned off, and gestures accompanying children's explanations were coded with the audio turned off. Children's explanations were coded in terms of the problem-solving strategies they conveyed, using the system described by Perry et al. (1988). Coding definitions and examples of the most commonly used correct and incorrect strategies are presented in Table 2. Additional details about coding procedures are provided in Alibali (1999).

Coding the relationship between gesture and speech—Finally, the relationship between gesture and speech in each response was evaluated. Any response that did not include gesture was coded as a *speech-alone* response. Any response in which speech and gesture expressed the same strategy was coded as a *gesture–speech match*. Any response in which gesture expressed one strategy and speech expressed a different strategy was coded as a *gesture–speech mismatch*.

Reliability

To assess reliability, 10% of the data was retranscribed and rescored by an independent coder, and we evaluated percentage agreement between the original coder and the reliability coder. Agreement was 100% for problem solutions, 91% for verbal solution strategies, 88% for gestured solution strategies, and 90% for the relationship between gesture and speech.

RESULTS

Do Children With SLI Show Impaired Accuracy on Addition and Equivalence Problems Relative to Children With Typical Development?

Addition problems—We used a one-way analysis of variance (ANOVA) to analyze accuracy on the addition problems across the three groups. As seen in Table 3, the groups differed in accuracy, with the children in the TD group outperforming the children in both

SLI groups, $F(2, 31) = 4.20$, $p < .05$, $\eta_p^2 = .21$ (i.e., 21% of the variation in performance was attributable to group membership). Because the data were not normally distributed in all groups (there was no variance in the TD group, with all children solving all three addition problems correctly), we also used a nonparametric test (chi-squared) to compare accuracy across groups. As seen in Figure 1, the proportion of children who correctly solved all three addition problems differed across groups, $\chi^2(2, N = 34) = 9.53$, $p < .05$.

These findings indicate that although the majority of 8- to 11-year-old children with SLI have good understanding of the concept of addition, their ability to correctly solve addition problems consisting of four numbers, all of which are addends < 7, is significantly impaired compared to that of their TD peers.

Equivalence problems—We also used a one-way ANOVA to analyze accuracy on the equivalence problems across the three groups. As seen in Table 3, the groups differed in accuracy, with the children in the TD group outperforming the children in both SLI groups, $F(2, 31) = 5.58$, $p < .05$, $\eta_p^2 = .27$. Because the data were not normally distributed in all groups (i.e., there was no variance in the ER-SLI group, with none of the children solving *any* of the equivalence problems correctly), we also used a nonparametric test (chi-squared) to compare performance across groups. As seen in Figure 2, the proportion of children who correctly solved *any* of the six equivalence problems also differed across groups, $\chi^2(1, N = 34) = 8.10$, $p < .05$. Comparing each of the SLI groups separately to the TD group, the proportion of children who solved any of the problems correctly differed for the ER-SLI and TD groups (0 of 8 vs. 10 of 17), $\chi^2(1, N = 25) = 7.84$, $p < .05$, but did not differ for the E-SLI and TD groups (3 of 9 vs. 10 of 17), $\chi^2(1, N = 25) = 1.53$, $p > .05$.

To gain some insight into how language abilities may influence children's performance, we looked at the performance of the children with SLI who successfully solved the equivalence problems on the tests of language ability and mathematical knowledge. The K-ABC scores for all three of the children in the E-SLI group who solved some of the equivalence problems correctly were ≥ 1 *SD* above age-level expectations (E-SLI 1, score 113; E-SLI 5, score 112; E-SLI 9, score 119). However, the two other children in the E-SLI group who had K-ABC scores ≥ 1 *SD* above age-level expectations (E-SLI 2, score 114; E-SLI 6, score 134) solved none of the equivalence problems correctly. Two of the three children in the E-SLI group who solved some of the equivalence problems correctly had RLSs at or above age-level expectations (E-SLI 1, RLS 120; E-SLI 5, RLS 101). The third child had the lowest RLS of children in the E-SLI group (E-SLI 9, RLS 89). Moreover, the other six children in the E-SLI group had RLS scores between 97 and 103 and solved none of the equivalence problems correctly. Additionally, one of the three children with E-SLI who solved some equivalence problems correctly had the highest ELS of the children in the E-SLI group (E-SLI 1, ELS 84). The other two had the third-highest score and the median score.

Of course, these analyses are exploratory, but they suggest that neither language ability nor mathematical knowledge is a definitive predictor of children's ability to solve equivalence problems. Correlational analyses of the TD group are also compatible with this conclusion.

In the TD group, performance on the CELF–R Oral Directions subtest (RLS) and number of equivalence problems solved correctly were uncorrelated, $r(17) = .08$, $p > .05$. However, not surprisingly, performance on the Kaufman Arithmetic subtest was significantly correlated with number of equivalence problems solved correctly, $r(17) = .72$, $p < .001$.

Do Children With SLI Use Different Strategies Than Children With Typical Development to Solve Equivalence Problems?

We next examined whether children with SLI and TD children used similar or different strategies to solve equivalence problems. Because different incorrect strategies lead to different solutions on the equivalence problems, examining children's solutions allows us to draw inferences about their understanding of the problems. The data are presented in Table 4, broken down for equivalence problems with the blank in final position (e.g., 6 + 5 + 4 = 6 + __) and those without the blank in final position (e.g., 5 + 6 + 2 = __ + 2). Within each group, children solved comparable numbers of blank-final and non-blank-final equivalence problems correctly.

With the exception of the other/unclassifiable category, for the problems solved incorrectly, the types of incorrect solution strategies used by the children were largely comparable across the three groups. The most common solution strategy for both groups was the add-all strategy (AA; which yields an answer of 15 for the problem 5 + 3 + 2 = 5 + __). This strategy also was used most often on blank-final problems in all of the groups, which is consistent with previous research with children with typical development (McNeil & Alibali, 2004).

The next most common strategy for all of the groups was the add-to-equal-sign strategy (AE; which yields an answer of 13 for the problem 5 + 6 + 2 = __ + 2). The AE strategy was used most often on non-blank-final problems for all of the groups. Interestingly, the AE strategy also was used by children in the SLI groups for blank-final problems, whereas children in the TD group used AE strategies only for non-blank-final problems. The combined add-all /add-to-equal-sign strategy (AA/AE; e.g., for the problem 5 + 6 + 2 = __ + 2, putting 13 in the blank and 15 at the end of the problem) was used infrequently, and only on non-blank-final problems in all three groups.

All of these strategies have been described in previous work on children with typical development (e.g., Alibali, 1999). They reflect misconceptions about equivalence problems that are commonly seen in children with typical development, and they have been attributed to gaps in children's understanding of the meaning of the equal sign symbol (McNeil & Alibali, 2005; Rittle-Johnson & Alibali, 1999). Only children in the SLI groups also provided solutions that did not fall into these categories or were unclassifiable.

Are Children With SLI Delayed in Their Acquisition of Equivalence, as Manifested in Their Gestured and Spoken Problem Explanations, Compared to TD Peers?

Finally, the sophistication of children's thinking about equivalence problems was examined using information from *both* their gestured and spoken explanations. In past research on children with typical development, individual children were “assigned” to a state along the path of the acquisition of mathematical equivalence (described in the Introduction; see also Alibali & Goldin-Meadow, 1993) based on each child's predominant response type. For purposes of these analyses, speech-alone responses were grouped with match responses, as in prior work (e.g., Alibali & Goldin-Meadow, 1993; Perry et al., 1988), because both types of responses convey a single strategy.

When children in this study were classified in this way, distinct differences among the groups emerged. As seen in Figure 3, the children with SLI were at earlier points along the developmental path than their TD peers. Importantly, the children in the ER-SLI group showed a more marked delay as compared to the children in the E-SLI group, who looked more similar overall to children in the TD group.

Are Children With SLI, and in Particular, Children With E-SLI, Especially Likely to Express More Advanced Knowledge About Equivalence in Their Gestures Than in Their Speech?

By definition, gesture–speech mismatch responses convey one strategy in speech and a different strategy in the accompanying gesture. Within any gesture–speech mismatch response, however, either both the gestured and verbal strategies can be incorrect, both can be correct, or one can be incorrect and the other correct. Responses in which speech conveys an incorrect strategy but gesture conveys a correct strategy are of particular interest because they represent responses in which gesture conveys *more advanced* information than speech.

Children's mismatch responses were classified into four categories: (a) response in which both gesture and speech conveyed different, incorrect strategies; (b) response in which gesture was more advanced than speech, in that gesture conveyed a correct strategy but speech conveyed an incorrect strategy; (c) response in which speech was more advanced than gesture, in that speech conveyed a correct strategy but gesture conveyed an incorrect strategy; and (d) response in which gesture and speech conveyed different, correct strategies.

Figure 4 presents the average proportion of mismatches in each category among children in each group who produced mismatches. As seen in the figure, the majority of mismatch responses produced by children in the ER-SLI group were ones in which both gesture and speech expressed incorrect strategies. In contrast, the majority of mismatch responses produced by children in the E-SLI group were ones in which gesture conveyed *more advanced* information than speech (e.g., a correct strategy in gestures and an incorrect strategy in speech). Finally, for children in the TD group, the majority of mismatch responses were ones in which both speech and gesture expressed correct strategies (i.e., two different correct strategies expressed in speech and in gesture).

These analyses reveal important differences between the ER-SLI and E-SLI groups. When children produced mismatches, the children in the ER-SLI group tended to express incorrect strategies in both modalities, whereas children in the E-SLI group more often expressed correct strategies in gestures with incorrect strategies in speech.

DISCUSSION

Empirical Summary

The purpose of this study was to investigate knowledge of mathematical equivalence in children with SLI using information they expressed in both speech and gestures. We hypothesized that children with SLI might have an appropriate concept of mathematical equivalence but might be likely to express their knowledge in gestures rather than speech.

Our findings replicate prior work indicating that children with SLI understand the concept of addition, but their ability to correctly solve addition problems is poor (e.g., Donlan, Cowan, Newton, & Lloyd, 2007). In the present study, even children with E-SLI who had nonverbal IQ and mathematical knowledge (as assessed by the Kaufman Arithmetic subtest) comparable to their TD peers had more difficulty correctly solving addition problems than children in the TD group.

The present findings extend prior work on mathematical knowledge in children with SLI by documenting difficulties in understanding of mathematical equivalence in children with SLI. Children with SLI were less likely than their TD peers to solve equivalence problems correctly. Analyses of Expressive, Receptive, and Kaufman scores for children with E-SLI did not reveal a clear pattern of skills that could account for children's performance on the mathematical equivalence problems. Children in all three groups made similar sorts of errors on the equivalence problems. The most common error involved adding all of the numbers in the problem; the next most common error involved adding the numbers up to the equal sign.

Data from children's speech and gestures were used to further characterize children's state along a developmental trajectory in the acquisition of equivalence that has been described in prior work with children with typical development (Alibali & Goldin-Meadow, 1993). The children with ER-SLI showed the most immature pattern, with all but one of the children in this group being at the beginning state of the developmental path. In this beginning state, children most often expressed a single, incorrect strategy for solving the equivalence problems in both gesture and speech. Children with E-SLI showed better understanding of the equivalence problems, with a few of the children in this group being at the final state of the path. In this final state, children most often expressed a single, *correct* strategy for solving the problems in both gesture and speech.

Moreover, when children with E-SLI produced gesture–speech mismatches, they often expressed correct strategies in their gestures but incorrect strategies in their speech. This pattern suggests that at ages 8–11, children with E-SLI may have an emerging understanding of equivalence that is represented in a nonverbal, perceptual format, which children cannot yet express in speech but which may become re-represented in an explicit, verbalizable format over developmental time.

Our findings raise the question of whether the performance of children with E-SLI and ER-SLI is suggestive of a delay or a different trajectory of development of understanding of mathematical equivalence, relative to children with typical development. On the whole, our findings suggest that children with E-SLI and those with ER-SLI exhibit delays in development. The solution errors made by children in both of the SLI groups were similar to those made by children with typical development. Further, the gestured and spoken problem explanations produced by children in both of the SLI groups suggest that most of these children were at the beginning of the path of acquisition of knowledge of equivalence—a state typical of younger children with typical development. Of course, future research will be needed to document whether children with SLI show the same pattern of progression along the developmental path as typical children, and in particular, whether children with SLI pass through a state in which they produce frequent gesture–speech mismatches in their problem explanations, as do most children with typical development (Alibali & Goldin-Meadow, 1993).

Most children with typical development show some success on equivalence problems by around age 10 (the end of fourth grade; McNeil, 2007). Our findings indicate that children with SLI showed delays in acquiring an understanding of mathematical equivalence, and children with ER-SLI showed greater delays in acquiring an understanding of mathematical equivalence than children with E-SLI. Indeed, none of the children in the ER-SLI group revealed an understanding of mathematical equivalence, even in their gestures. The children with ER-SLI also showed poor performance on ordinary addition problems, and they scored lower than their peers with E-SLI on a standardized test of mathematics knowledge (the Kaufman Arithmetic subtest). Taken together, these findings suggest that children with ER-SLI have substantial deficits in mathematical knowledge. Future studies should more directly investigate whether the mathematical difficulties experienced by children with ER-SLI are conceptual or computational in nature.

What Factors Might Account for Delayed Acquisition of Knowledge of Equivalence in Children With SLI?

The view that mathematical notation is akin to language suggests one possibility for why the children with SLI displayed delayed acquisition of knowledge of equivalence. Children need to learn that the equal sign has a particular meaning (i.e., *the same as*). Studies of children with typical development have shown that difficulties solving equations are associated with incomplete or incorrect knowledge of the semantic meaning of the equal sign (Knuth et al., 2006). Thus, it may be the case that the difficulties we observed in children with SLI are related to incomplete or incorrect knowledge of the semantic meaning of the equal sign—an indication that language skill may be related to mathematical knowledge, and specifically to the ability to understand the semantics of mathematical notation. Another, related possibility is that children with SLI may lack semantic knowledge of *same as*. However, this is unlikely because none of the children had any difficulty with initial judgments of equality on Piagetian conservation tasks, suggesting that children's difficulties were not due to a lack of understanding of the nonmathematical verbal concept of equality.

If language skill is related to the ability to understand mathematical notation, then why did children with E-SLI perform less well than children in the TD group on the equivalence task, given that the performance of children in these two groups on the Oral Directions subtest of the CELF–R did not differ? One possibility is that there is no direct causal relationship between language abilities and mathematical notation, but instead a third variable mediates the difficulties in both areas. Two recent studies have systematically investigated whether group differences in various number skills are explained by nonverbal IQ, working memory and short-term memory, language comprehension, vocabulary, and type of instruction the children received (Cowan, Donlan, Newton, & Lloyd, 2005; Koponen, Monomen, Räsänen, & Ahonen, 2006). Although these factors explained individual differences, in both studies, the group differences between children with SLI and typically developing peers were not accounted for by these factors. Interestingly, Koponen et al. (2006) found one significant predictor of group differences in numeracy skill—rapid serial naming. Recent work has shown that, relative to children with typical development, children with SLI show deficits in implicit serial learning of regularities across different modalities, including the visual–motor (Tomblin, Mainela-Arnold, & Zhang, 2007) and auditory modalities (Evans, Saffran, & Robe-Torres, 2009).

Deficits in implicit learning can lead to delays in acquisition of knowledge about mathematical equivalence because implicit learning is implicated in children's learning about the structure of equations (McNeil & Alibali, 2004). For example, studies of children with typical development have shown that children's extensive experience with equations that have the equal sign and answer blank at the end (e.g., 3 + 4 + 5 = ___) sometimes leads them to mis-encode equivalence problems as “ordinary” addition problems (e.g., 3 + 4 + 5 = 3 + ___ as 3 + 4 + 5 + 3 = ___) (McNeil & Alibali, 2004), and they consequently often add all the numbers in blank-final problems—a pattern we also observed in the present study. With greater exposure to “atypical” equations (e.g., 3 + 4 = 5 + 2, 3x + 4 = 13), as occurs in late elementary and middle school, children with typical development eventually begin to extract key structural regularities (e.g., the position of the equal sign) and correctly represent equations. If children with SLI have poor implicit learning abilities, they may require more exposure than their TD peers to mathematical equations in order to successfully extract the structural regularities that exist in equivalence problems and begin to encode equations correctly. Future studies should consider contributions of implicit learning in explaining difficulties in mathematical skills in children with SLI.

Limitations

Some important limitations of this study must be acknowledged. Most saliently, our sample size, particularly in the SLI subgroups, precluded use of some statistical tests and prevented us from drawing strong conclusions. Future studies are needed to replicate our main finding of a delay in acquisition of knowledge of equivalence in children with SLI, and to shed further light on differences in mathematical thinking between children with E-SLI and those with ER-SLI.

Second, our analyses focused on children's accuracy and their gestured and verbal problem explanations; we did not gather data about solution times. Additional data about timing would help to shore up our conclusion that children with SLI demonstrate delays in their development, rather than a different trajectory of development. Thus, we recommend that future studies incorporate analyses of solution times into their design and analysis.

Third, because the E-SLI and ER-SLI groups differed in multiple ways (e.g., receptive language, expressive language, nonverbal IQ), it was impossible for us to ascertain exactly what aspect of children's abilities was responsible for the group differences we observed. At first glance, the IQ differences between the groups seem to be a plausible source of the observed differences. However, it is important to note that all children in both of the SLI groups had IQs in the normal range. Furthermore, recent studies suggest that children with SLI with low IQ and those without low IQ do not differ, for example, in their response to intervention (Fey, Long, & Cleave, 1994). Therefore, although we cannot rule out IQ differences as a contributor, we believe that IQ differences are unlikely to account fully for the group differences we observed. Instead, we suggest that differences in receptive language are a more likely source of the observed differences between the two SLI groups. Other studies (e.g., Bishop, Adams, & Rosen, 2006) have shown that children with receptive deficits are less likely to benefit from interventions. It stands to reason that children with receptive deficits may be less likely to benefit from mathematics instruction in classroom settings, which can be construed as a form of "intervention" as well.

Educational Implications

Despite these limitations, our findings suggest some implications for the mathematics education of children with SLI. Math educators should be aware that children with language impairments may take more time or more practice to acquire a full-fledged understanding of mathematical concepts and procedures. In particular, educators should bear in mind that learning about mathematical notation may pose special difficulties for children with SLI. Children with SLI may need more repetitions of a mathematical symbol in context before acquiring its meaning. Explicit teaching about the meaning of mathematical symbols may also be beneficial.

Regarding mathematical equivalence more specifically, there is mounting evidence that appropriate understanding of the equal sign is crucial for success at the transition from arithmetic to algebra (e.g., Knuth et al., 2006). Given the importance of mathematical notation and symbol manipulation in algebra, the transition to algebra is likely to be challenging for children with SLI. Teachers should be aware that deficits in understanding mathematical equivalence may be a stumbling block for children who struggle to learn algebraic concepts and procedures.

Our findings further suggest that teachers may be well advised to consider the gestures as well as the speech that children produce when they talk about their knowledge, particularly for children with language impairments. Children's gestures may reveal emerging knowledge that is ready to be made more explicit, and that is an appropriate target for direct instruction (Alibali, Flevares, & Goldin-Meadow, 1997; Goldin-Meadow & Singer, 2003).

Finally, it is possible that the gestures that *teachers* use in instructional settings may be particularly important for children with SLI. Recent studies of children with typical development indicate that teachers' gestures contribute in important ways to children's mathematics learning (Goldin-Meadow, Kim, & Singer, 1999; Valenzeno, Alibali, & Klatzky, 2003). In particular, teachers appear to use gestures as a means of scaffolding students' understanding of complex mathematical concepts (Alibali & Nathan, 2007). Consistent with this view, a recent study of mathematics word problem solving in children with SLI yielded suggestive evidence that teachers' gestures can serve to scaffold children's understanding (Samelson, 2009).

Conclusion

Our main finding is clear: Children with SLI showed impaired performance on mathematical equivalence problems relative to TD peers. Moreover, children with ER-SLI showed greater deficits in performance than children with E-SLI. In considering potential sources of the observed patterns, we highlight the potential relevance of semantic interpretations of mathematical notation and of implicit learning, as discussed above. Future studies that directly address these issues are needed.

The present findings converge with past work (e.g., Evans et al., 2001; Mainela-Arnold et al., 2006) to suggest that gestures are an important window on understanding in children with SLI. Our data on children's gestured and spoken problem explanations suggest that children with SLI are at an earlier point in their acquisition of mathematical equivalence than their TD peers with typical development. Further, based on the gesture–speech data, the children with ER-SLI are at an earlier point in their acquisition of equivalence than their peers with E-SLI. Moreover, when children with E-SLI produced gesture–speech mismatches, they often expressed more advanced knowledge in their gestures than in their speech. Thus, focusing on speech alone in cognitive studies of children with SLI may at times lead to underestimation of their knowledge.

In sum, the present study extends past work on mathematical skills in children with SLI, revealing difficulties in understanding mathematical equivalence. The present findings suggest that at ages 8–11, children with ER-SLI are at the earliest developmental point in their acquisition of mathematical equivalence. In contrast, for 8- to 11-year-old children with E-SLI, gesture–speech mismatches often reveal a more advanced developmental pattern—knowledge that would have been missed had speech alone been assessed. Thus, gesture and speech together provide a more complete picture of knowledge about mathematical equivalence in children with SLI than speech alone.

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Appendix

APPENDIX

MATHEMATICAL PROBLEMS CHILDREN SOLVED

Addition

2 + 4 + 5 + 2 = __

3 + 6 + 4 + 3 = __

4 + 5 + 6 + 4 = __

Equivalence

Blank-final problems

5 + 3 + 2 = 5 + __

6 + 5 + 4 = 6 + __

3 + 4 + 6 = 3 + __

Non-blank-final problems

5 + 6 + 2 = __ + 2

6 + 4 + 5 = __ + 5

3 + 5 + 4 = __ + 4

Problem order

3 + 4 + 6 = 3 + __

5 + 6 + 2 = __ + 2

2 + 4 + 5 + 2 = __

5 + 3 + 2 = 5 + __

6 + 4 + 5 = __ + 5

3 + 6 + 4 + 3 = __

6 + 5 + 4 = 6 + __

3 + 5 + 4 = __ + 4

4 + 5 + 6 + 4 =__

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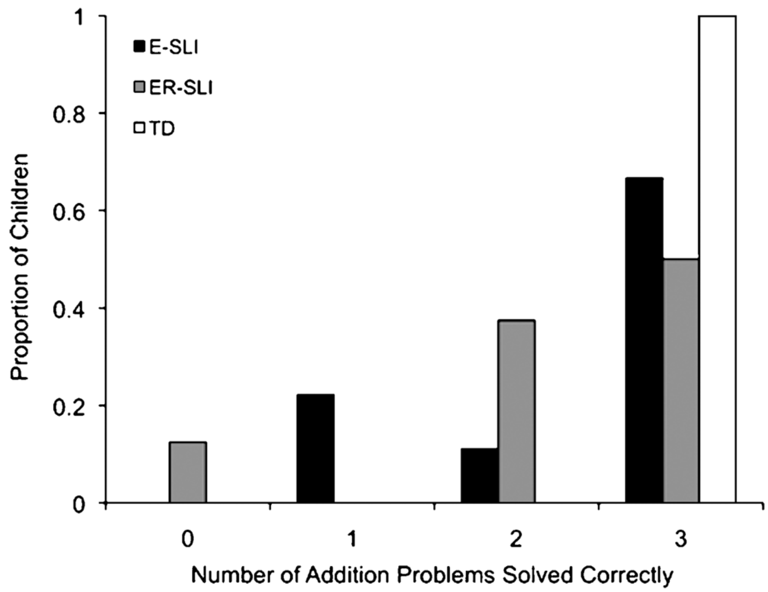


Figure 1.

Proportion of children in the group with expressive specific language impairment (E-SLI), the group with expressive and receptive SLI (ER-SLI), and the typically developing (TD) group who correctly solved 0, 1, 2, or 3 addition problems correctly.

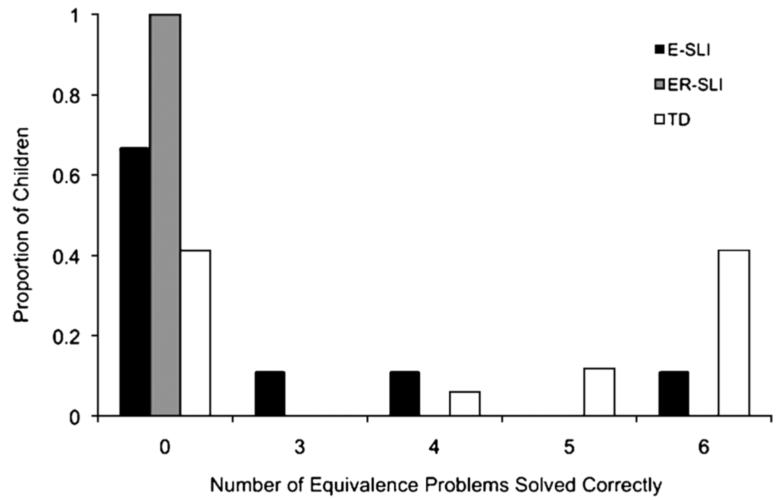


Figure 2.
Proportion of children in the E-SLI, ER-SLI, and TD groups who correctly solved 0, 3, 4, 5, or 6 equivalence problems correctly.

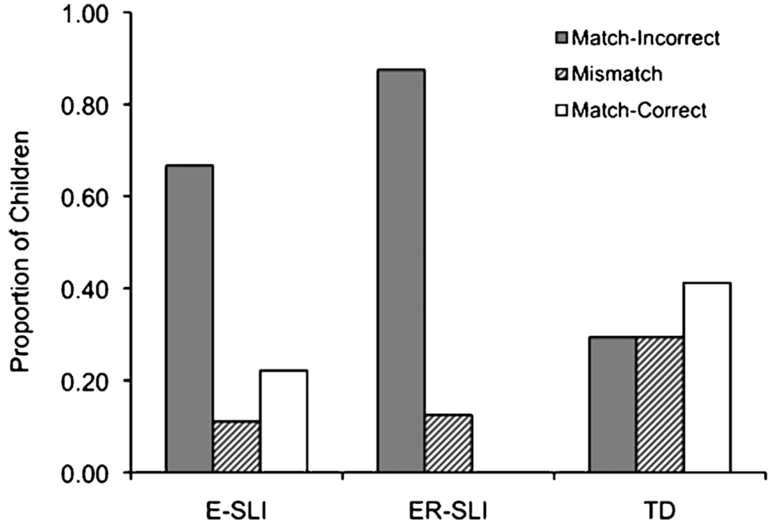
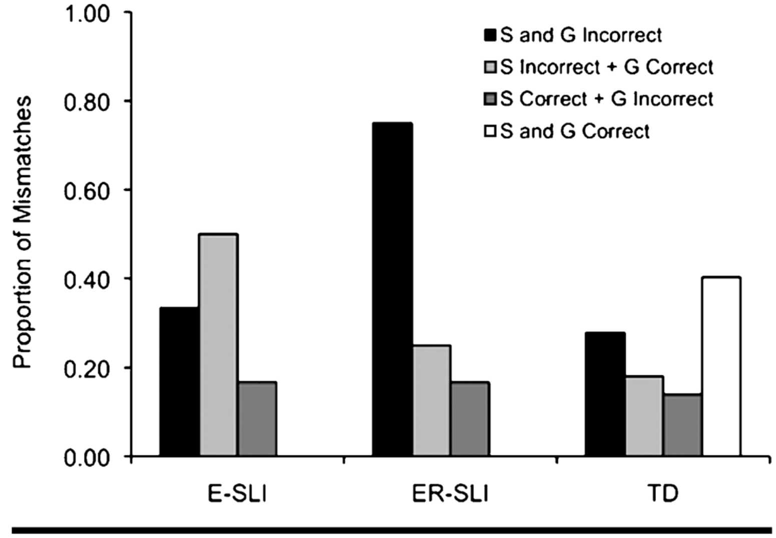


Figure 3.

Proportion of children in the E-SLI, ER-SLI, and TD groups at each point along the developmental path of acquisition of mathematical equivalence.



Note. S = speech, G = gesture.

Figure 4.

Average proportion of mismatches of each type produced by children in the E-SLI, ER-SLI, and TD groups who produced mismatches.

Table 1

Age and standardized test scores for the group with expressive specific language impairment (E-SLI), the group with expressive and receptive SLI (ER-SLI), and the typically developing (TD) group.

	Age in months	ELS[a]	OD[b]	RLS[c]	K-ABC[d]	IQ[e]
E-SLI						
Mean	119.89	78.44*	9.67*	100.00*	109.78*	111.00*
SD	13.75	3.84	2.23	8.88	12.68	8.80
Range	97–139	72–84	6–13	89–120	94–134	95–120
ER-SLI						
Mean	116.88	69.38*	5.00*	71.13*	86.63*	100.25*
SD	12.33	8.96	1.69	10.53	7.48	10.22
Range	102–135	62–86	3–8	54–83	73–96	85–118
TD						
Mean	118.24	104.71*	11.12**		117.88**	118.13**
SD	12.12	10.69	2.47		10.48	14.33
Range	99–138	88–124	8–15		94–133	93–141

[a]Expressive Language score of the Clinical Evaluation of Language Fundamentals—Revised (CELF-R; Semel, Wiig, & Secord, 1989; $M = 100$, $SD = 15$).

[b]Oral Directions receptive subtest score of the CELF–R ($M = 10$, $SD = 3$).

[c]Receptive Language score of the CELF–R ($M = 100$, $SD = 15$).

[d]Kaufman Assessment Battery for Children score (Kaufman & Kaufman, 1983; $M = 100$, $SD = 15$).

[e]Intelligence Quotient as measured by the Leiter International Performance Scale (Roid & Miller, 1997), the Test of Nonverbal Intelligence, Third Edition (Brown, Sherbenou, & Johnsen, 1997), or the Columbia Mental Maturity Scale (Burgemeister, Blum, & Lorge, 1972; $M = 100$, $SD = 15$).

*$p < .05$, two tailed, groups differ.

**$p < .05$, two tailed, TD and ER-SLI groups differ, but TD and E-SLI groups do not.

Table 2

Examples of strategies commonly expressed in speech and gesture.

Strategy	Definition	Spoken explanation	Gestured explanation (handshape: referents)
Incorrect			
Add all	Add all the numbers in the problem. In gesture: Indicate all numbers in the problem without differentiating the addend on the right side from the others.	(6 + 5 + 4 = 6 + _)“6 plus 5 is 11 plus 4 is 15 plus 6 is 21.”	(3 + 4 + 6 = 3 + _) RH point: touch left 3, 4, 6, right 3
Add to equal sign	Add the numbers on the left side. In gesture: indicate all numbers on the left side.	(3 + 5 + 4 = _ + 4) “3 and 5 is 8. And plus 4 is 12.”	(6 + 4 + 5 = _ + 5) LH point: left 6, 4, 5, solution
Correct			
Grouping	Group the numbers that do not appear on both sides of the equation. May also note that the other number appears on both sides. In gesture: Indicate the numbers that do not appear on both sides. May also indicate the number that appears on both sides, but in a way that is differentiated from the others (i.e.. in a different gesture unit, marked by putting hand down or by using a different handshape).	(3 + 5 + 4 = _ + 4) “The fours were taken care of, and 3 plus 5 is 8.”	(3 + 5 + 4 = _ + 4) LH & RH points: left and right 4s; LH & RH palms: cover left and right 4s; LH palm: indicate 3, 5
Add–subtract	Add the numbers on the left side of the equation and subtract the number on the right. In gesture: Indicate all numbers on the left side, then indicate the number on the right side in a way that is differentiated from the others (i.e., in a different gesture unit, marked by putting hand down or by using a different handshape).	(3 + 4 + 6 = 3 + _) “I took 3 plus 4, which is 7, and added it to 6, then that makes 13, so I took 3 off 13 and got 10.”	(5 + 6 + 2 = _ + 2) LH open hand: sweeps under 5 + 6 + 2 =, hand down; RH open hand: touches right 2, hand down; RH open hand: touches solution
Equalize	Make both sides of the equation sum to the same total. In gesture: indicate all numbers on the left side, then indicate all numbers on the right side in a way that is differentiated from the left side (i.e., in a different gesture unit, marked by putting hand down or using a different handshape).	(6 + 4 + 5 = _ + 5) “There's 6 plus 4 is 10 plus 5 is 15, so 10 plus 5 would be 15.”	(6 + 4 + 5 = _ + 5) RH pinky point: left 5, 6, 4, left 5, hand down; RH point: solution, R5

Note. RH = right hand, LH = left hand.

Table 3

Number of addition and equivalence problems solved correctly by children in the E-SLI, ER-SLI, and TD groups (*SD* in parentheses).

Group	Addition *N* = 3 problems	Equivalence *N* = 6 problems
E-SLI	2.44 (.88)	1.44 (2.30)
ER-SLI	2.25 (1.04)	0.00 (0)
TD	3.00 (0)	3.29 (2.89)

Table 4

Percentage of correct and incorrect solution strategies used by children in the E-SLI, ER-SLI, and TD groups for blank-final and non-blank-final equivalence problems.

Group and problem type	Correct solutions	Incorrect solutions			
		AA	AE	AE/AA	Other/UC
E-SLI					
Blank-final	22	56	4	0	17
Non-blank-fmal	26	41	19	7	6
ER-SLI					
Blank-final	0	88	4	0	13
Non-blank-final	0	38	42	13	13
TD					
Blank-final	53	47	0	0	0
Non-blank-fmal	57	27	10	6	0

Note. AA = add all; AE = add to equal sign; AE/AA = add to equal sign + add all; Other/UC = other or unclassifiable.